Cascade and breakdown in scale-free networks with community structure

Jian-jun Wu,[1,2,*] Zi-you Gao,[1,2,†] and Hui-jun Sun[1,2]
[1]*State Key Laboratory of Rail Traffic Control and Safety, Beijing Jiaotong University Beijing, 100044, People's Republic of China*
[2]*School of Traffic and Transportation, Beijing Jiaotong University Beijing, 100044, People's Republic of China*
(Received 22 May 2006; published 19 December 2006)

The cascade failures in scale-free networks with community structure are studied and cascade propagation of such networks with different modularity parameters is simulated. It is found that the network with small modularity is much easier to trigger cascade failures than that of the larger one. Furthermore, different removal strategies have some what large effects on the cascade failures aftereffect. The simulations also show that larger modularity and reserve capacity coefficient will delay the breakdown caused by a cascade of network. This is particularly important for such real networks with community as traffic networks, distribution networks, and electrical power grids.

DOI: 10.1103/PhysRevE.74.066111 PACS number(s): 89.75.Hc

A lot of interest has been focused on the characterization of various structural and locational properties of networks, such as degree distribution, clustering coefficient, small world effects and so on. Among the others, an important property common to many networks is the presence of community structure. Community structure means many networks in nature or social networks can be divided into some groups such that the connections within each group are dense, while connections between groups are sparse. For instance, in social networks some individual can be part of a tightly connected group or of a closed social elite, others can be completely isolated, while some others may act as bridges between groups [1]. In very great degree, structure determines the characteristic and function of networks [2]. The finding of community structure provides a powerful tool for understanding the growth mechanisms and the functioning of the complex network.

In addition, the behaviors and dynamics in complex networks have recently attracted the attention of researchers in different areas. Among them we recall the robust against random failure and intentional attacks of nodes or edges, the effect of cascade, etc. Cascade failures are initiated when a heavily loaded node (edge) is lost for some reason, and the load on that node (edge) (i.e., the flow passing through it) must be redistributed to other nodes (edges) in the network. This redistribution may cause other nodes (edges) to exceed their capacity causing them also to fail. Hence the number of failed nodes (edges) increases, propagating throughout the network. In particularly serious cases the entire network is affected. Cascading failures have been observed in many real complex networks. The largest blackout in U.S. history took place on 14 August 2003, a typical example of cascading failure in electrical power grids [3]. Resistance of networks to the removal of nodes or edges, due either to random breakdowns or to intentional attacks, has been studied in Refs. [4–8]. Such studies have focused only on the static properties of the network showing that the removal of a group of nodes altogether can have important consequences [9]. However, there are few works about the cascade in scale-free networks by considering the community structure. In this paper, based on the recently addressed problem of "community structure" and "cascade" in complex networks, we intend to fill this gap by proposing a cascade failure model based on the scale-free networks with community structure, and the effects of cascade with community structure is studied.

*Electronic address: wjj419@sohu.com
†Electronic address: gaoziyou@jtys.bjtu.edu.cn

I. GENERATE SF NETWORKS WITH COMMUNITY STRUCTURE

Scale-free networks can be generated with various methods. The first model is given by Albert and Barabási (BA) [10] which they identify as being necessary for a network to have the scale-free property, (i) *growth*—the network grows over time; (ii) *preferential attachment* (PA)—nodes with a high degree are more likely to create edges to new nodes than ones with a low degree.

Inspired by these two mechanisms mentioned above and the previous work by Yan [11], we propose the generation model as follows: Starting with c communities denoted as $U_1, U_2, \ldots, U_c$. And for each community with a small number (m_0) of initial nodes, at every time step we add into each community a new node with m ($<m_0$) edges that link the new node to n ($n<m$) different nodes within this community and m-n different nodes within other $c-1$ communities already present in the system according to PA rule, $\Pi(k_i)=k_i/\Sigma k_j$ which means the probability Π that the new node will be connected to node i depends on the degree k_i of vertex i. In our study, we find the equilibrium status of systems is asymptotic stability [12]. And the result shows that the scaling behavior is independent on its initial topology (the number of nodes m_0) and the variation of m and n value. So the results will not change qualitatively following changes in the parameters of the network generation. Additionally, the degree distribution $p(k)$ of nodes of the local network (inside each community), as well as the global network have stronger stabilities in the growing process with PA growing mechanism and follow the power law with exponent 3, i.e., $p(k) \propto k^{-3}$, (see Fig. 1).

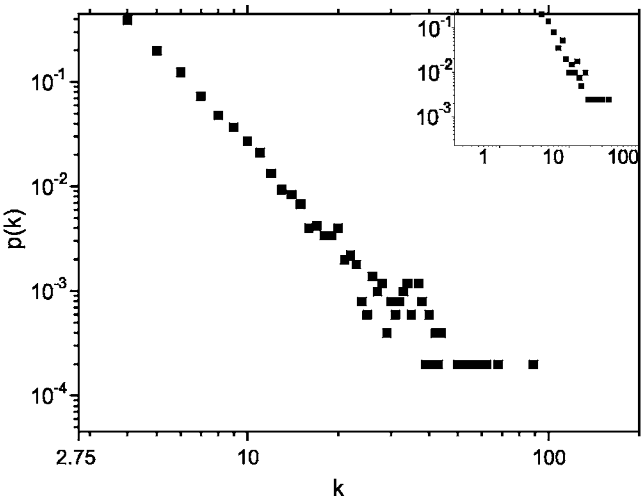

FIG. 1. Double-logarithm global-degree and local-degree (inset) distribution of a network with $N=520$, $m_0=m=4$, $n=1$, and $c=4$. The graph corresponds to the average 30 realizations of the networks.

Based on a previous measure of assortative mixing [13], Newman and Girvan proposed a measure of the quality of a particular division of a network, which they called the modularity as follows [14]: $Q=\Sigma_{\mathrm{r}}(h_{rr}-a_r^2)$, where $a_r=\Sigma_w h_{rw}$ denotes the row (or column) sums which represent the fraction of edges that connect to nodes in community r and h_{rw} is the fraction of edges in the original network that connect nodes in subset r with nodes in subset w. In a given network in which edges fall between nodes without regard for the communities they belong to, $h_{rw}=a_r a_w$ can be obtained. In our network model, we can adjust the c value to get networks with various strength Q of community structure. The larger the value of Q is, the most accurate a partition into communities will be. If the number of within-community edges is no better than random, we will get $Q=0$. Values approaching $Q=1$, which is the maximum, indicate strong community structure [14]. In this paper, the edge whose two nodes are in a same community is defined as the local edge and the bridge edge represents the edge whose two nodes belong to different communities.

II. CASCADE FAILURE MODEL BASED ON SF NETWORKS WITH COMMUNITY STRUCTURES

In our cascade failure model, each edge is assigned with a given capacity according to the betweenness of edges to handle the traffic. Initially the network is in a stationary state in which the load at each edge is smaller than its capacity. Removing an edge will change the balance of load and leads to a redistribution of loads over other edges. This process is called the initial attack. A redistribution of loads will trigger the overload failures of other edges and eventually a large drop in the network performance which is called the propagation of cascade. The main differences of our work with respect to previous models [9,15–17] are as follows: (i) The removal strategies are divided as inner removal (removal of the local edge with the maximum load) and inter removal (removal of the bridge edge with the maximum load). (ii) When the cascade arrives at the stationary state, a new traffic flow is added into the network in order to make heavier damage, attacking the network leads to a new cascade; iteration of this process until the breakdown of network. (iii) Not only the decrease in the network efficiency [18], but also the change of average shortest path $\langle D\rangle$ defined as the mean of geodesic lengths over all couples of nodes [19] is proposed to quantify the damage caused by a cascade.

The urban transit and road network are found to display the properties of scale free networks [20,21]. Therefore, here we represent an urban traffic network as undirected graph $G=(V,K)$ where V is the set of nodes, and K is the set of edges. G is described by $N\times N$ the adjacency matrix $\{e_{ij}\}$. We define N as the size of the network. Then, the network efficiency is [17]

$$E(G)=\frac{1}{N(N-1)}\sum_{i\neq j\in G}\frac{1}{d_{ij}},$$

where d_{ij} indicates the efficiency of the most efficient path between i and j.

In a real traffic network, each edge has a design capacity denoted as $L_{ij}(0)$ according to the load between origin i and destination j. For simplicity, the load $L_{ij}(t)$ on the edges between i and j at time t is assigned depending on the betweenness of this edge $\sigma_{ij}(t)$, where $\sigma_{ij}(t)$ is defined as the total number of most efficient paths passing through the edge e_{ij} at time t. Following Ref. [15], we also assume the maximum handle capacity H_{ij} of edge e_{ij} is proportional to its design capacity $L_{ij}(0)$. That is:

$$H_{ij}=\alpha L_{ij}(0),$$

where α is a tolerant parameter. In traffic systems, by setting traffic control methods, the actual load passing the edges will be the multiples of the design capacity, and this multiple is called reserve capacity coefficient [22]. Here, α is a tolerant parameter, which corresponds to the reserve capacity coefficient.

III. SIMULATION RESULTS

We performed a numerical simulation with a total of $N=120$ nodes, and a given scale-free network with community generated by BA. That is, at each time step, we connect a new node with three nodes in a selected community and connect it to a node in another community. The selection of size $N=120$ is based on two reasons: (i) This relatively small system can be seen as simulating the backbone of a city's urban traffic network, and this does provide a sufficiently sized network to gain statistically significant attributes; (ii) The whole cascade process will cost too much running time of a computer in order to make the traffic system breakdown. The evolving network with our model can be seen from Fig. 2.

Here we focus on a cascade triggered by two removal strategies: Inter removal and inner removal. Furthermore, the influence on the cascade damage of a network with different modularity is studied.

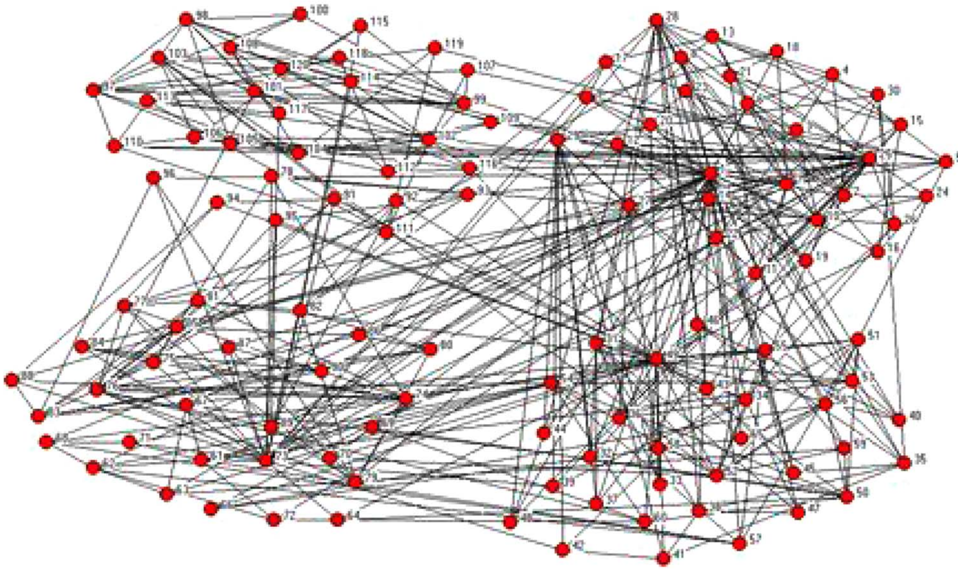

FIG. 2. (Color online) A simple scale-free network with communities generated by our model. In this case there are four communities, $N=120$, $m_0=m=4$, $n=1$, and $c=4$.

In our paper, we obtain the different modularity $Q=0.14,0.15,0.16$ by adjusting the parameters m and n over 15 average ($Q=0.14$: $m=3$, $n=1$; $Q=0.15$: $m=4$, $n=1$; $Q=0.16$: $m=6$, $n=1$). From Fig. 3, we can see that the efficiency decreases under cascade and attacking, while the average shortest path increases initially and then decreases. For a larger value of modularity Q, the efficiency is also higher, however, the average shortest path keeps opposite when the time step is small. With the time step increase, the average shortest path become higher for the larger value of modularity. Figure 3 also shows that larger modularity can delay the phase transition time and then delay the breakdown time of the network. For different modularity $Q=0.14,0.15,0.16$ in Fig. 3(a), the evolving time of network breakdown trigged by a cascade are about 420, 580, and larger than 600, respectively. And from Fig. 3(b), we can also see that the larger jumps for inter removal occur at 220, 300, and 520 steps. But for inner removal, it occurs at 200, 280, and 440 time steps. In the simulation, the breakdown of network will be delayed more than 100 time steps if the modularity increase is 0.01, which can be understood easily. The higher the modularity is, the smaller the fraction of edges among communities in total edges will be, and the more homogeneous the network is. Therefore, the scale-free characteristics become more inconspicuous within a community, and cascade failures in a small modularity network more easily happen. This is consistent with some recent works on cascade failures in complex networks based on different dynamical models [15], where it has been shown that cascade failures are much less likely to happen in a homogeneous network than in a heterogeneous network. Therefore, the network with larger modularity will take on a good robustness and resistance to the damage of a cascade. Additionally, the same result can be draw for different removal strategies. Figure 3 also indicates that different removal strategies have a small effect on the efficiency of a network, but bring large changes of the average shortest path. We know the efficiency is the sum of the effective path. Only a few edges are connected among communities, so the impact on the efficiency for two removal

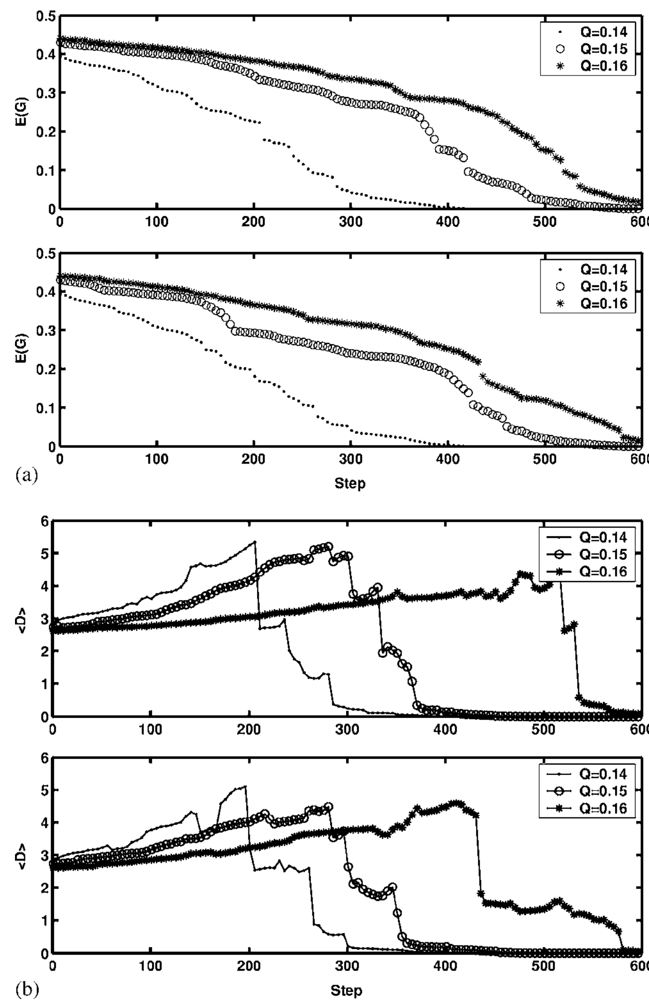

FIG. 3. The global efficiency $E(G)$ (a) and the average shortest path $\langle D\rangle$ (b) of the network after cascading and attacking, as a function of the evolving step with different modularity for different removal strategies (for every graph, the top one corresponds to the inter removal and bottom one represents the inner removal), respectively.

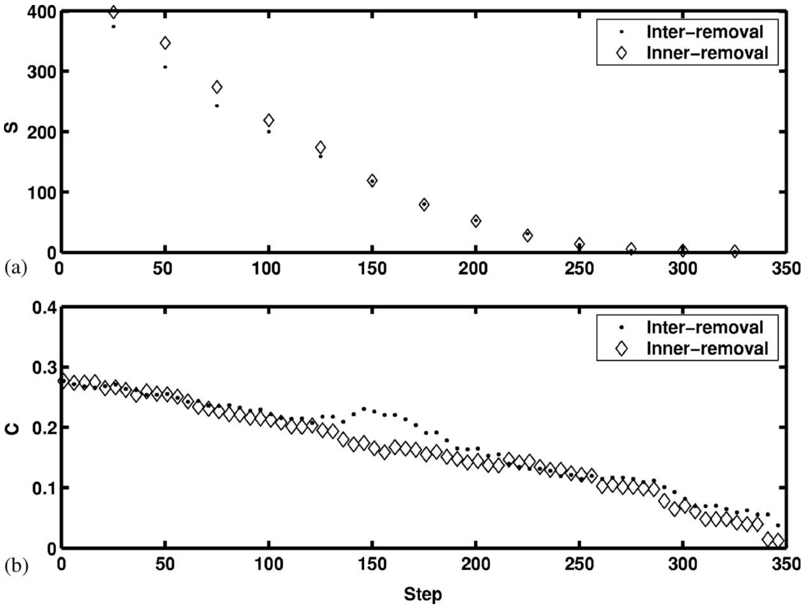

FIG. 4. The changes in the size of the largest cluster S (a) and the clustering coefficient C (b) as a function of step (time) for inter removal (dot) and inner removal (diamond). The graph corresponds to the average 30 realizations of the networks.

strategies is almost equivalency, and the modularity will determine and affect the function of the network.

When edges are removed from a network, clusters of nodes whose links to the system disappear may be cut off (fragmented) from the main cluster. To better understand the impact of attacks on the network structure, we investigate this fragmentation process. We measure the size of the largest cluster, S, shown as a fraction of the total system size, when a fraction of the edges are removed either inner removal or inter removal. For both inner removal and inter removal S decreases after the network starts to break down, eventually S changes as a function of step as shown in Fig. 4(a). One can see that for the inner removal the breakdown is slower compared to the inter removal. Another important element is average clustering coefficient C. As seen in Fig. 4(b), C is a decrease with the time evolution, and the variety for two removal strategies is not very obvious. When time steps exceed 150, Fig. 4 indicates that two removal strategies have the same largest cluster. The rough functional forms are quadratic and linear for S and C, respectively ($S=3.3t^2-81t+490$ and $C=-0.000\,71t+0.28$).

Different modularity can display different structures and functions of many real-world networks. Figure 5 is the time evolving of modularity for two removal strategies. One can see that the modularity tends to decrease with the evolution of time. Furthermore, there have small fluctuations during the evolving process. The reason is that, at time t, the modularity will decrease if the local edge is removed caused by a cascade. However, the removal of the bridge edge will lead to the increase of modularity. Compared with inter removal, inner removal will cause a drop of the modularity. The rough functional form of Fig. 5 for the modularity and time steps is cubic. Actually, we also obtain approximate fitting function by experiments as follows: $Q=-2.15\times10^{-9}t^3+8.84\times10^{-8}t^2-0.000\,13t+0.146$.

Figure 6 shows the variety of efficiency as a function of the reserve capacity coefficient α in the cascading and breakdown of a scale-free network with communities for inter removal and inner removal, respectively. Clearly, for two cases, we can see that the damage is smaller for larger values of α. Small efficiency indicates the network is difficult to realize its function. Therefore, Fig. 6 also indicates the larger α value can delay the breakdown of the network. Different removal strategies will cause the variety of efficiency, especially for larger value of α. But for the same α value, the efficiency decreases with the evolving time. Additionally, the damage of two removal strategies is almost equivalent when $\alpha=1.05$, 1.25, and 1.5. Another finding is that when the cascade step is small, the effects of reserve capacity coefficient α on efficiency are not clear. But with the time evolving, its effects become more and more obvious. For a given time step (i.e., $t=200$ or 300), when $\alpha=1.5$, the efficiency of network can be increased 13% compared with $\alpha=1.05$. Therefore, the finding is very important for the traffic network especially for the urban jamming road networks. For example, we can adjust the reserve capacity coefficients based on the real traffic status to avoid the whole network collapse

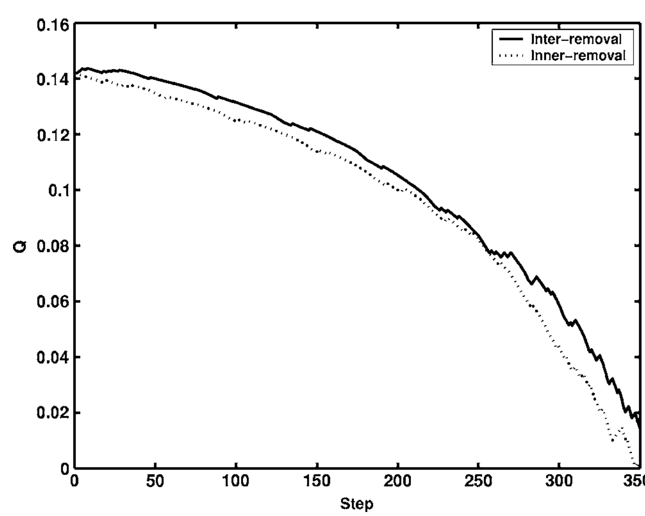

FIG. 5. The time evolving of modularity for two removal strategies (inter removal and inner removal). The value α is 1.3 in the simulation.

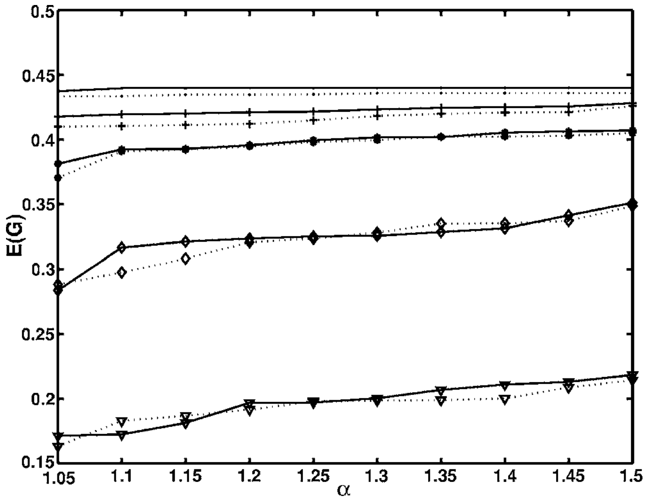

FIG. 6. The changes of $E(G)$ as a function of reserve capacity coefficient α for inter removal (dashed line) and inner removal (solid line) in different cascade steps (dot: step=5; plus: step=50; star: step=100; diamond: step=200; inverted triangle: step=300).

triggered by a congested route. This result is significantly important for the design of traffic networks.

In this paper we simulate the cascade failures in scale-free network with community structure, in which the local degree and global degree all obey power-law distributions. Additionally, we focus on the study of cascade with different removal strategies, inter removal and inner removal, to understand the influence of the cascade in such networks. Our result is thus that different removal strategies on a single important edge and different modularity of network may trigger different cascades of overload failures capable of disabling the network almost entirely. Furthermore, the network becomes more vulnerable to the small modularity and reserve capacity coefficient. These results suggest that to avoid cascade failures in the scale-free network with community, large modularity and reserve capacity coefficients are feasible. We hope this work might shed some light on the analysis and control of cascade failures and its propagation in real-world complex networks.

ACKNOWLEDGMENTS

This paper was partially supported by National Basic Research Program of China (Grant No. 2006CB705500), National Outstanding Young Investigator (Grant No. 70225005), NSFC of China (Grant Nos. 70501005, 70471088, and 70631001), Natural Science Foundation of Beijing (Grant No. 9042006), the Innovation and Special Researcher Foundation of Beijing Jiaotong University (Grant Nos. 48006 and 48111).

[1] S. Fortunato, V. Latora, and M. Marchiori, Phys. Rev. E **70**, 056104 (2004).
[2] J. J. Wu, Z. Y. Gao, H. J. Sun, and H. J. Huang, Europhys. Lett. **74**, 560 (2006).
[3] J. Glanz and R. Perez-Pena, *90 Seconds That Left Tens of Millions of People in the Dark* (New York Times, New York August 26, 2003).
[4] R. Albert, H. Jeong, and A.-L. Barabási, Nature (London) **406**, 378 (2000).
[5] P. Holme, B. J. Kim, C. N. Yoon, and S. K. Han, Phys. Rev. E **65**, 056109 (2002).
[6] P. Crucitti, V. Latora, M. Marchiori, and A. Rapisarda, Physica A **320**, 622 (2003).
[7] M. Girvan and M. E. J. Newman, Proc. Natl. Acad. Sci. U.S.A. **99**, 8271 (2002).
[8] A. E. Motter, T. Nishikawa, and Y. C. Lai, Phys. Rev. E **66**, 065103 (2002).
[9] P. Crucitti, V. Latora, and M. Marchiori, Phys. Rev. E **69**, 045104(R) (2004).
[10] A.-L. Barabási and R. Albert, Science **286**, 509 (1999).
[11] G. Yan, Z.-Q. Fu, J. Ren, and W.-X. Wang, e-print physics/0602137.
[12] H. J. Sun and J. J. Wu, Technical Report of Institute of Systems Science, Beijing Jiaotong University, 2006 (unpublished).
[13] M. E. J. Newman, Phys. Rev. E **67**, 026126 (2003).
[14] M. E. J. Newman and M. Girvan, Phys. Rev. E **69**, 026113 (2004).
[15] A. E. Motter and Y. C. Lai, Phys. Rev. E **66**, 065102(R) (2002).
[16] Y. Moreno, J. B. Gomez, and A. F. Pacheco, Europhys. Lett. **58**, 630 (2002); Y. Moreno, R. Pastor-Satorras, A. Vazquez, and A. Vespignani, *ibid.* **62**, 292 (2003).
[17] R. Cohen, K. Erez, D. ben-Avraham, and S. Havlin, Phys. Rev. Lett. **85**, 4626 (2000); **86**, 3682 (2001).
[18] V. Latora and M. Marchiori, Phys. Rev. Lett. **87**, 198701 (2001).
[19] D. J. Watts and S. H. Strogatz, Nature (London) **393**, 440 (1998).
[20] J. J. Wu, Z. Y. Gao, H. J. Sun, and H. J. Huang, Mod. Phys. Lett. B **18**, 1043 (2004).
[21] M. Rosvall, A. Trusina, P. Minnhagen, and K. Sneppen, Phys. Rev. Lett. **94**, 028701 (2005).
[22] Z. Y. Gao and Y. F. Song, Transp. Res. **36B**, 319 (2002).